UNIT-2

(Lecture-5)

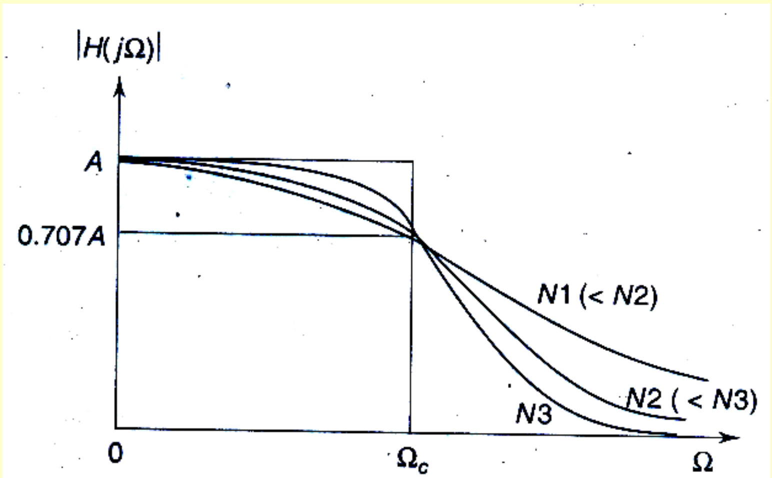
Design of Infinite Impulse Response Digital Filters: Butterworth Filters

Raj Ranjan Prasad

Butterworth Filters

The Butterworth low pass filter has a magnitude response given by $H(j\Omega| = \frac{A}{\left[1+(\Omega/\Omega_c)^{2N}\right]^{0.5}}$ ----------(1)

Where A is the filter gain and Ω_c is the 3dB cut-off frequency and N is the order of the filter. The magnitude response of the Butterworth fitter is shown in figure.



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Butterworth Filters

The magnitude response has a maximally flat pass-band and stop-band. It can be seen that by increasing the filter order N, the Butterworth response approximates the ideal response. However, the phase response of the Butterworth filter becomes more non-linear with increasing N.

The design parameters of the Butterworth filter are obtained by considering the low-pass filter with the desired specifications as given below.

$$\delta_1 \leq |H(e^{j\omega})| \leq 1, \; 0 \leq \omega \leq \omega_1 \qquad \text{---------(2a)}$$

$$|H(e^{j\omega})| \leq \delta_2, \; \omega_2 \leq \omega \leq \pi \qquad \text{---------(2b)}$$

Butterworth Filters

Using eq. (1) in eq. (2) and if A=1, we get

$$\delta_1^2 \le \frac{1}{1+(\Omega_1/\Omega_c)^{2N}} \le 1 \quad \text{---------(3a)}$$

$$\frac{1}{1+(\Omega_2/\Omega_c)^{2N}} \le \delta_2^2 \quad \text{---------(3b)}$$

Eq.(3) can be written in the form

$$(\Omega_1/\Omega_c)^{2N} \le \frac{1}{\delta_1^2} - 1 \quad \text{---------(4a)}$$

$$(\Omega_2/\Omega_c)^{2N} \ge \frac{1}{\delta_2^2} - 1 \quad \text{---------(4b)}$$

Assume equality and divide eq.(4b) by eq. (4a)

$$(\Omega_2/\Omega_1)^{2N} = \left(1/\delta_2^2 - 1\right)/\left(1/\delta_1^2 - 1\right) \quad \text{---------(5)}$$

$$N = \frac{1}{2}\,\frac{\log\left\{[(1/\delta_2^2 - 1)/(1/\delta_1^2 - 1)]\right\}}{\log\,(\Omega_2/\Omega_1)} \quad \text{----------(6)}$$

Butterworth Filters

The value of N is chosen to next nearest integer to the value of N.

Using eq. (4a) , we get

$$\Omega_c = \frac{\Omega_1}{\left[(1/\delta_1^2) - 1\right]^{1/2N}} \quad \text{--------(7)}$$

The value of Ω_1 and Ω_2 are obtained using $\Omega = 2/T\,[\tan(\omega/2)]$ for bilinear transformation or $\Omega = \omega/T$ impulse invariant transformation.

Butterworth Filters

The transfer function of the Butterworth filter is usually written in the factored form as given below

$$H(s) = \prod_{k=1}^{N/2} \frac{B_k \, \Omega_c^2}{s^2 + b_k \, \Omega_c \, s + c_k \, \Omega_c^2} \qquad N = 2, 4, 6, \ldots$$ ----------(8)

or,

$$H(s) = \frac{B_0 \, \Omega_c}{s + c_0 \, \Omega_c} \prod_{k=1}^{(N-1)/2} \frac{B_k \, \Omega_c^2}{s^2 + b_k \, \Omega_c \, s + c_k \, \Omega_c^2} \qquad N = 3, 5, 7, \ldots$$ ------(9)

The coefficients b_k and c_k are given by

$$b_k = 2 \sin [(2k-1)\pi/2N] \text{ and } c_k = 1$$ ------(10)

Butterworth Filters

The parameter B_k can be obtained from

$$A = \prod_{k=1}^{N/2} B_k \text{, for even } N \quad \text{----------------(11)}$$

$$A = \prod_{k=1}^{(N-1)/2} B_k \text{, for odd } N \quad \text{----------------(12)}$$

The system function of the equivalent digital filter is obtained from H(s) (eq.8 or eq.9) using the specified transformation technique (bilinear transformation or Impulse invariant transformation)